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Simulation of the light evolution in an annular photobioreactor for the cultivation of *Porphyridium cruentum*

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HIGHLIGHTS

- An integrated model for the cultivation of microalgae including flow, radiation and microorganism growth is presented.
- The evolution of light transfer in the photobioreactor (PBR) is well captured both in the batch and continuous cultures.
- The radiation of two polychromatic light sources is successfully predicted by the box model and corrected light/dark ratio.

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ABSTRACT

Light availability inside the reactor is often the bottleneck in microalgae cultivation and the light intensity varies with its position and time in the cultivation process. An integrated model including flow, radiation and microorganism growth is presented, in which the radiation of two complementary polychromatic light sources is resolved with the finite volume method combined with a box model. The integration of the box model into radiative transport equation (RTE) is verified first and then utilized to predict the microalgae concentration evolutions in a batch and continuous culture, respectively, which are in a good agreement with the experimental data. The evolution of light transfer in the photobioreactor (PBR) is well captured in both cultures, which provides a guideline to promote the light utilization in the PBR. The model developed and verified in this contribution has the potential to be applied as an effective tool to scale up these types of reactors and achieve an optimal biomass production with the precise control of the cultivation.

1. Introduction

Microalgae are a new natural resource with a lot of potential industrial interests, but their production under controlled conditions requires a dedicated reactor called photobioreactors (PBRs), which differ from classic bioreactors, fermentors or enzymatic reactors, mainly by the need of a light supply in addition to classic chemical growth substrates (Pruvost et al., 2002). The culture productivity is invariably controlled by the availability of light, particularly as the scale of operation increases (Molina Grima et al., 1999; Cuaresma et al., 2011). The light attenuation in a PBR is a function of the cell concentration and the light absorption characteristics of the cellular pigments (Chrismadha and Borowitzka, 1994), while the light intensity distribution is generally non-uniform inside the reactor (Yang et al., 2004). The culture volume in a PBR can indeed be delimited schematically into two zones, namely an illuminated zone where photosynthetic activity is higher than respiration (resulting in the specific growth rate $\mu > 0$), and a dark zone where respiration is higher ($\mu < 0$). These zones may have different volumes during the process of culture cultivation. The time span that the cells reside in a specific zone is a function of the culture fluid-dynamics (Grobbelaar et al., 1996). Moreover, hydrodynamics conditions are proposed to affect the light conversion in PBRs, by modifying the light availability of suspended photosynthetic cells (Pruvost et al., 2008). Although great progresses have been achieved in modeling the hydrodynamics and radiation in PBRs (Pruvost et al., 2002; Huang et al., 2010, 2011), and a great deal of work has been done to develop PBRs for algal cultures, more efforts are still needed to improve PBR technologies and understand the growth mechanism of the algal culture (Ugwu et al., 2008).

A simple Lambert–Beer law has been widely adopted to predict the radiation in PBRs (e.g., Janssen et al., 2000; Suh and Lee, 2003; Benson et al., 2007; Bosma et al., 2007; Elyasi and Taghipour, 2010 and Li et al., 2010). However, it is inappropriate to model the light intensity in PBRs with this oversimplified model in most cases.

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First, the model is correct only in dilute solutions with the monochromatic light and the light absorption independent of cells (Suh and Lee, 2003; Li et al., 2010). Second, the model cannot be used to predict the radiation distribution in an annular PBR since the lamps emit photons neither from one point source nor in parallel (Rosello Sastre et al., 2007; Imoberdorf and Mohseni, 2011). Finally, the contribution of scattered photons to neighboring volume elements is ignored in this model, while this effect should be taken into account to predict the distribution of photon flux density (PFD) properly. Even for simple geometries the errors of applying the Lambert–Beer law are often large, especially, when the scattering coefficient is big (Rosello Sastre et al., 2007). Moreover, the analysis of the PBR system based on the local available light energy presents a valid means of determining the algal cell growth rate (Suh and Lee, 2003). In recent years, the discrete ordinate method (DOM) and the finite volume method (FVM) have emerged as the two most attractive methods for modeling radiative transfer mainly due to their high accuracy, wide applicability and relatively low computational cost and computer memory requirement (Huang et al., 2011).

It is well known that photosynthesis is limited to wavelengths between 400 nm and 700 nm (photosynthetically active radiation, PAR), and microalgae is usually cultivated with polychromatic light source (Muller-Feuga et al., 2003; Berberoglu et al., 2007). However, it is very difficult to model the distribution of radiation in PBR accurately with polychromatic light due to the fact that the absorption coefficient and the scattering coefficient are both spectrally dependent. Berberoglu et al. (2007) predicted the one-dimensional steady radiation transfer in a plane-parallel PBR with a great success using a box model where the spectral dependence of the radiation was considered. However, to the best of our knowledge, there is no other published literature in predicting the polychromatic light radiation in PBRs, especially for complex structures with a two-dimensional or three-dimensional geometry. However, such occasions are frequently encountered in practice.

In addition to the effect of radiation, the period of the light/dark cycle is also important in microalgae growth. The fraction of time that microalgae spend in the illuminated zone versus the dark zone is defined as the light/dark ratio. It is evident that microalgae are out of the light field when they are in the dark zone inside the reactor or in the accessory equipments out of the reactor, during which the negative growth rate of the culture incurs negative influence on the performance of the PBR. As time evolves, the concentration of the culture increases and the dark zone in the reactor gradually becomes larger implying the light/dark ratio is time dependent. Since the photosynthesis rate increases linearly as the light intensity increases in the weak illumination zone (Luo and Al-Dahhan, 2004), how to define the light/dark ratio under such circumstance is a nontrivial problem. To the best of our knowledge, there are currently no studies in the available literature to predict the evolution of light in the process of cultivation, let alone a model to compute the light/dark ratio for these complex situations.

In this contribution, an integrated model including the multi-field coupling of flow, radiation and microorganism growth is presented to predict the evolution of cell concentration in the time course of cultivation. The FVM developed by Chai (1994) is adopted in present contribution to discretize the governing equation due to its favorable characteristics especially that it allows for conserving the radiant energy (Huang et al., 2011). Additionally, the box model applied by Berberoglu et al. (2007) is used to predict the two-dimensional radiation with polychromatic lights in annular chambers. Furthermore, the effect of dark zone in the PBR on the light/dark ratio is examined and a quantitative method is proposed. The focus of this contribution is to validate the box model integrated with the radiative transport equation (RTE) in the simulation of the polychromatic light transfer in a PBR and illustrates its practical applications in the cultivation of *Porphyridium cruentum* in the batch and continuous regimes, respectively. It is shown that the variation of light intensity in the PBR is non-linear and spatiotemporal. This work gives a clear insight into the evolution of the light intensity in the PBR and provides valuable information for the design and optimization of the PBR for a specific application.

2. Mathematical models and basic assumptions

The basic RTE with polychromatic radiation for an absorbing and scattering medium at the position $\vec{r}$ in the direction $\vec{s}$ can be written as (Hostikka and McGrattan, 2006; Jean-François, 2010):

$$\frac{dI_\lambda(\vec{r},\vec{s})}{ds} = -a_\lambda I_\lambda(\vec{r},\vec{s}) - \sigma_\lambda I_\lambda(\vec{r},\vec{s}) + \frac{\sigma_\lambda}{4\pi}\int_0^{4\pi} I_\lambda(\vec{r},\vec{s}')\Phi_\lambda(\vec{s}\cdot\vec{s}')d\Omega' \quad (1)$$

This equation indicates that the light intensity depends on the spatial position and angular direction. The sum of the absorption coefficient and the scattering coefficient is often called the extinction coefficient:

$$\beta_\lambda = a_\lambda + \sigma_\lambda \quad (2)$$

The incident intensity at any spatial position from all the directions is given by

$$G_\lambda(\vec{r}) = \int_{\Omega=0}^{\Omega=4\pi} I_\lambda(\vec{r},\vec{s})d\Omega \quad (3)$$

For polychromatic radiation, integration over the wavelength (frequency) range of interest must be performed. Hence, the total local irradiance can be calculated with the following summation (Pottier et al., 2005; Berberoglu et al., 2007):

$$G(\vec{r}) = \sum_{\lambda_{\min}}^{\lambda_{\max}} G_\lambda(\vec{r}) \quad (4)$$

Then, the total instantaneous local irradiance can be used to predict the microorganism growth.

The simple Markov chain is chosen as the growth model, so that the culture concentration at time $t+\Delta t$ only depends on its concentration at the preceding time t (Muller-Feuga et al., 2003). If the time step Δt is divided into two periods, i.e., the light period ($\omega\Delta t$) and the dark period ($(1-\omega)\Delta t$)) where ω is the light/dark ratio discussed below, the change of the microorganism concentration in the zone of illumination can be calculated by

$$C_{t+\omega\Delta t} = C_t e^{(\mu-D)\omega\Delta t} \quad (5)$$

When the culture is in the dark zone without any light, only the respiration exists and the photosynthetic activity stops. Under this circumstance, the specific growth rate reaches its negative peak, resulting in the decrease of the microorganism concentration, which can be written as

$$C_{t+\Delta t} = C_{t+\omega\Delta t} e^{(\mu-D)(1-\omega)\Delta t} \quad (6)$$

The growth of the culture can be obtained by using a biological growth model that gives the growth rate as a function of the received light intensity. Although some novel mechanistic growth models for phytoplankton have been proposed to represent the physiology of the photosynthetic cells in recent years (Eilers and Peeters, 1988; García-Camacho et al., 2012), the model parameters are difficult to derive for a specific species, and their effectiveness requires further verification (Luo and Al-Dahhan, 2004). Instead, a simple phenomenological model for the growth of *P. cruentum* is adopted here as follows (Pruvost et al., 2002;

Muller-Feuga et al., 2003)

$$\mu = \frac{2\mu_s(I_s - I_c)(I - I_c)}{(I_s - I_c)^2 + (I - I_c)^2} \tag{7}$$

This model expresses the photosynthetic response of microalgae to the instantaneous light available in the illuminated zone. In the accessory equipments out of the reactor where the light intensity is zero, the specific growth rate can be calculated by Muller-Feuga et al. (2003)

$$\mu = \frac{2\mu_s(I_c - I_s)I_c}{(I_s - I_c)^2 + I_c^2} \tag{8}$$

The fluctuating light history induced by the flow can modify the instantaneous conversion rate of the absorbed light. It is called the light–dark (L/D) cycle effect (Janssen et al., 2000). The light/dark ratio for each loop denoted as ω is given by

$$\omega = t_l/t_{\text{total}} = 1 - t_d/t_{\text{total}} = 1 - (t_{rd} + t_{au})/(t_r + t_{au}) \tag{9}$$

All the parameters in the above equation are constant except the time in the dark zone inside the reactor (t_{rd}), which should be corrected (subsequently called the corrected L/D ratio) in the time course of the microorganism growth and will increase as the time elapses when the phenomenon of photolimitation happens. It is not a trivial work to calculate the duration time that the cells reside in the dark zone inside the reactor in each loop. Fortunately, the volume-averaged irradiance is equivalent to the time-averaged value when the cells are distributed homogenously throughout the reactor (Luo and Al-Dahhan, 2004). To quantify the light/dark cycling time, one needs to know the relative volumes of the photic and dark zones, as well as the velocity of the fluid interchange between these zones (Molina et al., 2000). In this situation, plug flow in the reactor is assumed and the entire liquid phase of the reaction medium is fully mixed by the propeller at the end of the loop. So the time in the dark zone inside the PBR can be simply estimated as follows

$$t_{rd} = t_r(V_{rd}/V_r) \tag{10}$$

The new concentration at time $t+\Delta t$ is determined by volumetrically averaging the whole concentrations in radial elementary volume. Calculation of the culture concentration in the dark zone is performed at the end of the fraction time step for the light part ($\omega\Delta t$). Since the microalgae concentration is low and the size of the microalgae is very small, isotropic scattering is supposed in this work. In most microalgae cultures, the concentration does not increase from the beginning and a lag time occurs. The initial acclimation lag period of 1.2 days, which is the same as the value taken by Muller-Feuga et al. (2003), is adopted in this work.

Table 1
The structure of the reactor and the operating parameters for batch and continuous cultures.

Categories	Parameter	Value
Geometry conditions	Inner radius of the reactor	20 mm
	Outer radius of the reactor	50 mm
	Chambers	8
	Length of the chambers	1500 mm
Flow conditions	Bulk velocity in light chambers	5 cm s^{-1}
	Time for one loop, t_{total}	5 min
	The duration in auxiliary equipments, t_{au}	1 min
	The duration in the reactor, t_r	4 min

3. Simulation conditions

The experimental data of Pruvost et al. (2002) with the Grolux lamp are chosen to evaluate the accuracy of the integration of the box model into RTE for modeling the transfer of polychromatic light with the method of FVM. The experimental data provided by Muller-Feuga et al. (2003) with batch and continuous cultures are chosen to compare with the predicted results and demonstrate the integrated model's practical applicability. In the experiments, the inoculation concentration is fixed at 0.06 g L^{-1} in both the batch and continuous cultures. The PBR consists of a reaction loop with eight annular geometries, i.e., light chambers, which are connected in series and provided with two alternative artificial lights. Each of the light chambers contains a central fluorescent tube that illuminates the culture flowing through the annular gap. All the chambers are 1.5 m long, with the internal and external diameters of 40 mm and 100 mm, respectively. The details of the geometry and operating conditions are summarized in Table 1.

The two alternating light sources of the PBR are 1.5 m long Grolux-type tubes and Satin-type tubes. These sources are chosen for their complementary stimulation of microalgae photosynthesis pigment. The light spectra of both lamps are given in Fig. 1 and these lamps deliver a mean photosynthetically active PFD of 175 μE m^{-2} s^{-1} and 236 μE m^{-2} s^{-1}, respectively (Pruvost et al., 2002).

As noted above, the absorption coefficient and the scattering coefficient both depend on the wavelength of the polychromatic light. In order to simplify the numerical simulations, the PAR is divided into multiple sections where the spectral quantities are estimated by using a box model (Modest, 2003; Berberoglu et al., 2007). This model approximates the spectral quantities using a series of boxes, whose area under the original spectrum equals to that under the box. Fig. 2 shows the extinction coefficients of *P. cruentum* determined by using a spectrophotometer (Pruvost et al., 2002) and

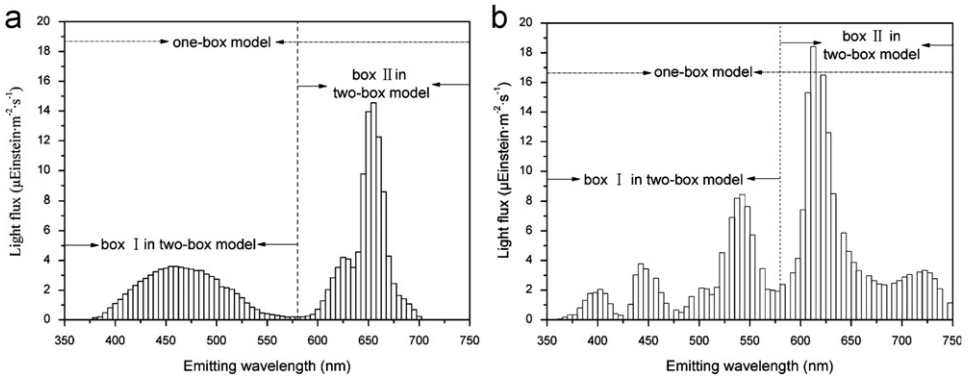

Fig. 1. Emitting spectra of both light sources and the application of box model: (a) Grolux-type tube and (b) Satin-type tube.

approximated with the box model. In case that only one box is used, the model is equivalent to a monochromatic radiation. Here, considering the computational cost, the characteristics of the emitting spectra of both lamps and the profile of the extinction coefficients with spectra, two boxes with the wavelength intervals from 350 nm to 580 nm and 580 nm to 750 nm are selected. Because there are two different light sources, the results by using the averaged light and the two different lights are compared as two different models in the batch and continuous cultures. The details of the box model employed in this work for the sources and the culture are illustrated in Figs. 1 and 2, with the associated parameters listed in Tables 2 and 3, respectively. Since there are no experimental profiles for the absorption coefficient and the scattering coefficient of microalgae with the polychromatic light, the corrected extinction coefficient is adopted as the absorption coefficient, similar to what has been applied by Pruvost et al. (2002) and Muller-Feuga et al. (2003), and the difference between the extinction coefficient and the corrected value can be assumed as the scattering coefficient. The corrected value with one box model in this work is approximately equal to 0.08 $m^2\,g^{-1}$, which was measured by Brindley et al. (2011) for microalgae.

Pruvost et al. (2008) argued that the separation between these two zones according to the specific growth rate defined the dark and illuminated regions, and the residence time in each region defined the periods of the light–dark (L/D) cycles. However, it is found that it will result in a large dark region in the reactor and a lengthy dark period from our preliminary simulations when the aforementioned parameter of the compensation light intensity in the growth model is adopted as the threshold to discern the light zone and the dark zone in the reactor. A systematic search of the parameter is therefore performed and a posteriori value of 0.05 $\mu E\,m^{-2}\,s^{-1}$, which is far below the compensation value of *P. cruentum*, is found to agree well with the experimental data and thus is adopted here. That means all the control volumes inside the reactor, whose light intensity is smaller than this value, will be summed up and identified as the dark zone. Although reflection in the reactor can be totally ignored due to the fast decay of light, the reflectivity coefficients of 0.9 and 0.2 for the quartz sheathe covered the lamps and the walls are used here, respectively (Huang et al., 2011). The parameters used in the biological growth model are the same as those in Muller-Feuga et al. (2003) and the details can be found in Table 4.

As indicated above, the absorption coefficients and the scattering coefficients are biomass concentration dependent, so all the optical coefficients are updated before the simulation of radiation. A sufficiently small time step of 5 min is applied here and the error due to time discretization can be safely neglected. Finally, grid and angular discretization sensitivity studies are performed to make sure that the computed values of the local spectral irradiance are independent of the grid size and control angle. It is found that the numerical error due to the discretization can be neglected when the solid angle, 4π steradians, is discretized into 8×16 uniform control angles in the polar and the azimuthal angle, and the uniform grid with 30×300 nodes in the radial and axial directions is chosen. A QUICK scheme with deferred-correction method (Huang et al., 2010) for the upstream boundary intensities of the control volume is adopted to avoid the numerical diffusion. All the simulations are performed using an in-house developed FORTRAN code in this work (Huang et al., 2011).

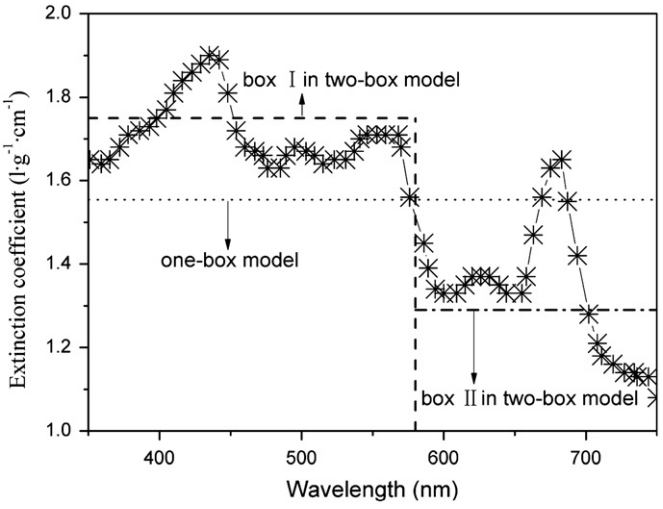

Fig. 2. The box model applied to the extinction coefficients of *P. cruentum*.

4. Results and discussion

4.1. Validation of the RTE with box model for polychromatic light transfer

In the experiments of Pruvost et al. (2002), the extinction coefficients were changed across a wide range of optical thickness by increasing the concentration of the culture. The transmitted light intensities in the middle of the light chamber's outer cylinder were measured and normalized. Comparisons of the predicted results using different models with the experimental data are illustrated in Fig. 3. It can be seen that the relative local light profiles predicted with different models agree very well with the experimental data and there are no significant differences among them. However, the two-box model is a little better than the one-box model and can be used as a useful tool for the prediction of light intensity inside the reactor under these conditions. It is noteworthy that although the integration of the two-box model into RTE presents similarly satisfactory results as the modified Beer–Lambert law method does, these two methods are substantially different in terms of absolute values. This is resulted from the fact that the modified Beer–Lambert law is only

Table 2
The parameters in the box model for the lamps.

Categories		Parameter	Value ($\mu E\,m^{-2}\,s^{-1}$)
One-box model		Average PFD	205.5
Two-box model	Grolux lamp	Total flux	175
		Wavelength less than 580 nm	75.8
		Wavelength greater than or equal to 580 nm	99.2
	Satin lamp	Total flux	236
		Wavelength less than 580 nm	91.9
		Wavelength greater than or equal to 580 nm	144.1

Table 3
The optical coefficients of the culture in the box model.

Categories	Variable	Value ($m^2\ g^{-1}$)
One-box model	Extinction coefficient with one box	0.1554
	Absorption coefficient with one box	0.06993
	Scattering coefficient with one box	0.08547
Two-box model	Extinction coefficient for wavelength less than 580 nm	0.175
	Absorption coefficient for wavelength less than 580 nm	0.07875
	Scattering coefficient for wavelength less than 580 nm	0.09625
	Extinction coefficient for wavelength equal or greater than 580 nm	0.129
	Absorption coefficient for wavelength equal or greater than 580 nm	0.05805
	Scattering coefficient for wavelength equal or greater than 580 nm	0.07095

Table 4
Parameter values used in the growth model.

Categories	Variable	Value
Culture conditions	Acclimation lag phase duration	1.2 d
	Dilution rate (D) for batch culture	0 d^{-1}
	Dilution rate (D) for continuous culture	0.23 d^{-1}
Growth conditions	Specific growth rate at saturation, μ_s	1.42 d^{-1}
	PFD at saturation, I_s	385 μE m^{-2} s^{-1}
	Compensation PFD, I_c	3.5 μE m^{-2} s^{-1}

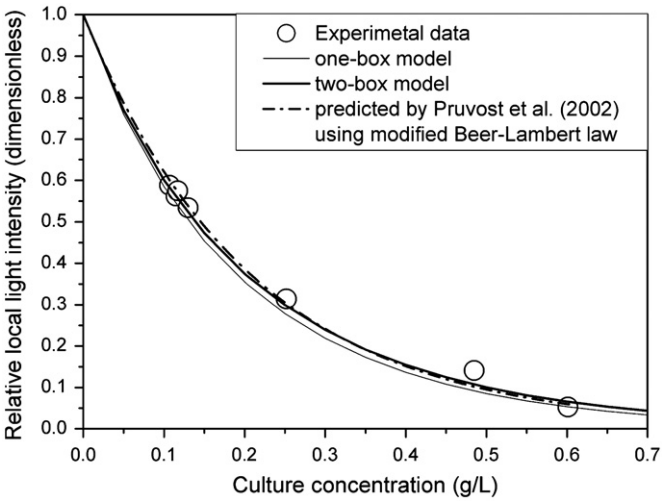

Fig. 3. Experimental values of relative local light intensities vs. predicted data under different culture concentrations.

applicable when the light source is a collimated one without scattering and reflection. When the culture concentration is zero, the modified Beer–Lambert law will result in a much bigger value due to neglect of the area difference between the source and the outer cylinder. Accordingly, it will give higher absolute values of light intensity when the culture concentration increases. That is to say, though similar relative profiles are obtained, the values derived from the modified Beer–Lambert law will always be much higher than those obtained from the two-box model in all the cases. Additionally, it is shown from the figure that the attenuation of light is a function of the instantaneous concentration of the cells and the local light intensity is time dependent in the whole process of the cultivation.

Typical distributions of the light intensity predicted in the PBR with the two-box model are presented in Fig. 4. It is clear from these figures that the distribution of light field in the reactor is non-uniform. Although the culture concentration is dilute, the attenuation of the light intensity is so quick that it decreases to a small value only just a few centimeters away from the source. This is the reason why the PBR is usually designed with a smaller optical path length. The differences of the light intensity among the middle, bottom and top parts of the PBR are due to the effect of reflections on the walls. It can also be concluded that the light availability is the limiting nutrient and the light utilization is the principal limiting factor affecting microalgae productivity (Richmond, 2004). Therefore, it has been widely accepted that the promotion of light utilization by an algal culture within the reactor is an effective way in the design and scaling-up of the PBRs.

4.2. Integration of the box model into RTE for batch culture

The radiation calculation in the PBR was performed by using an averaged light combined with one box model, two lights integrated with one box model and two lights in combination with two boxes model, respectively. The growth of the culture is obtained by using the biological growth model described above with and without the correction of the L/D ratio. Comparison of the calculated microalgae concentrations in the process of *P. cruentum* cultivation with experimental data by Muller-Feuga et al. (2003) is shown in Fig. 5.

It can be seen from Fig. 5 that all the models capture the change of culture concentration very well in the first 15 days. However, the discrepancy occurs in the later stage. The experimental curve is accurately predicted by combining the two lights model with the two boxes model and the corrected L/D ratio. It also shows that the influence of the selection of box model on the simulation results is critical, especially in the last 15 days of cultivation. It can be deduced from the results of these models with two lights that the light intensity predicted with the two boxes model is slightly stronger than that of the one box model, which results in a higher predicted culture concentration in the late period of cultivation. Although the model with two lights and two boxes is more reasonable in the acquisition of light intensity in principle, the growth curve can be well predicted only if the L/D ratio for the growth model is properly considered. The specified value demarcating the dark zone is much smaller than the compensation light

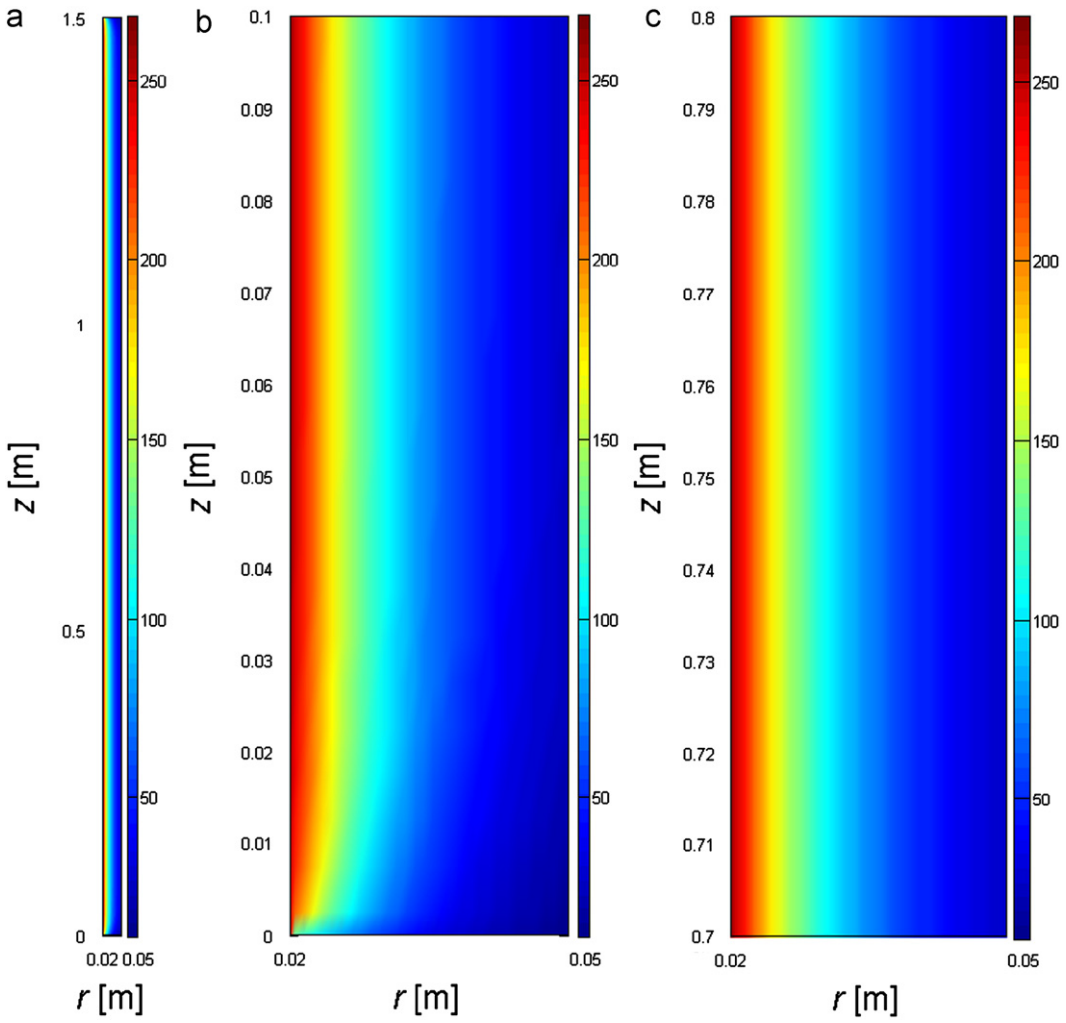

Fig. 4. Distribution of the predicted local light intensity inside the reactor with the culture concentration of 0.3 g L^{-1}. (a) overview; (b) partial enlarged view at the bottom; (c) partial enlarged view in the middle.

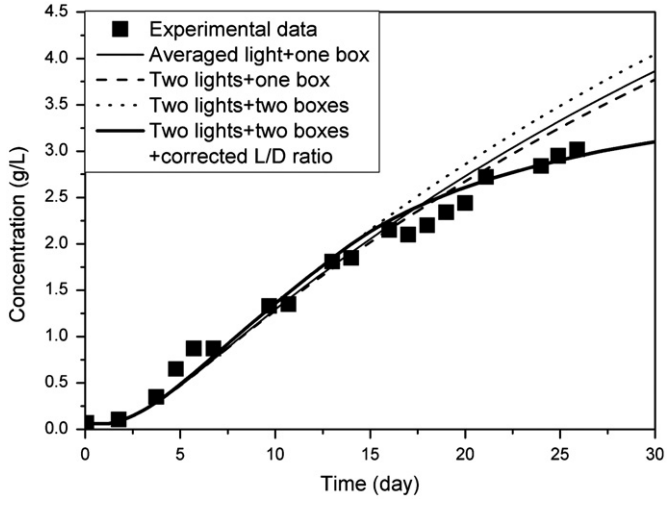

Fig. 5. Comparison of the predicted concentrations with experimental data by Muller-Feuga et al. (2003) for batch cultures of *P. cruentum*.

intensity in this contribution. The reason might be the fact that the photosynthetic activity still takes place in the weak illumination zone where the light intensity ranges from zero to that just below the compensation light intensity, though the respiration dominates in this region. This zone therefore cannot be taken as the traditional dark one without any light at all. Hence, a corrected value is chosen in this contribution to quantitatively describe the effect of insufficient light on the growth of cells.

The evolutions of the light intensity in the middle of the PBR with two light sources in the whole process of cultivation are depicted in Figs. 6 and 7. These figures illustrate that the light intensity distributions of these two light sources are similar to each other in the whole process of cultivation despite different absolute quantities. Under this scenario, the radiation field in the PBR is not uniform, and the light intensity is spatiotemporally dependent. Obviously, light is adequate near the sources; the decay of light is fast and a large portion of the volume, which is far away from the sources, is extremely short of light only five days after the inoculation. It can be concluded from these results that special external light source is demanded to complement the decay of light, which is important to the microalgae cultivation.

The specific growth rates in the radial direction of the PBR computed using Eq. (7) with two light sources during the time course are presented in Fig. 8. It is shown that photolimitation phenomenon is again observed in both conditions only five days after the initiation of algal cultivation in most volume of the PBR. Negative growth rates are observed at some locations far away from

the lamps in the later period of cultivation. That is to say, mass reduction occurs in these zones because the respiration is higher than the photosynthetic activity. This is compatible with the radiation distributions in Figs. 6 and 7. The specific growth rate at the same location decreases slowly after 15 days of cultivation.

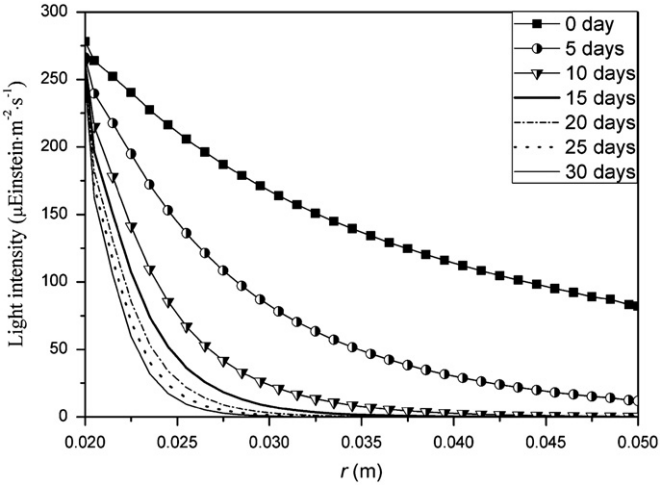

Fig. 6. Evolution of the light intensity with Grolux lamp in *P. cruentum* culture.

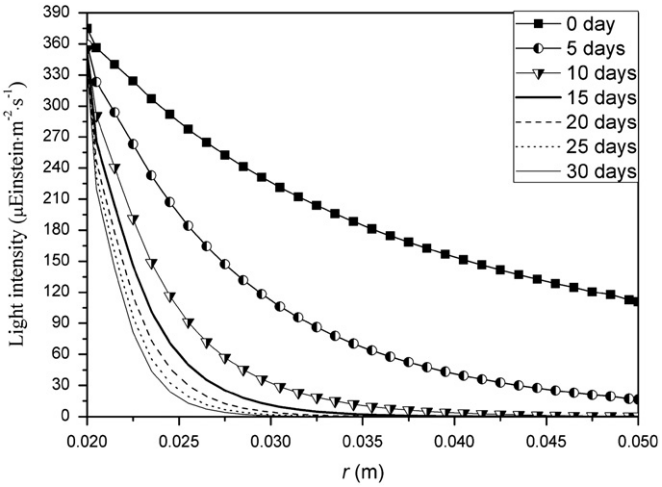

Fig. 7. Evolution of the light intensity with fluorescent lamp in *P. cruentum* culture.

4.3. Integration of the box model into RTE for continuous culture

The concentrations of the microorganism predicted with averaged light associated with one box, two lights combined with one box or two lights with two boxes and the experimental values as a function of time in a continuous mode are illustrated in Fig. 9. Compared to the batch culture, the phase of exponential growth is evident in this case. The predicted data with the model of two lights combined with two boxes agree well with the experimental data and are much better than other models. However, the difference between simulations and experiments is slightly bigger during the period of exponential growth in all the models, and the underestimation is probably due to the inaccuracy of the specific growth rate models. The simulation results of the model with two lights combined with two boxes and corrected L/D ratio are not presented here and will be discussed later.

The evolution of the light intensity in the reactor predicted with the model of two lights and two boxes is shown in Figs. 10 and 11. The distribution of the light intensity in the PBR in this case is also similar to that of the batch mode in the same condition. However, the decay of light is slower in the reactor compared to that of the batch mode due to the dilution of culture. Although the distribution of the light intensity is better, it is far away from the ideal situation and a large portion of the volume is useless in most of time in the process of cultivation. It is also demonstrated that the light supply is the main constraint in this

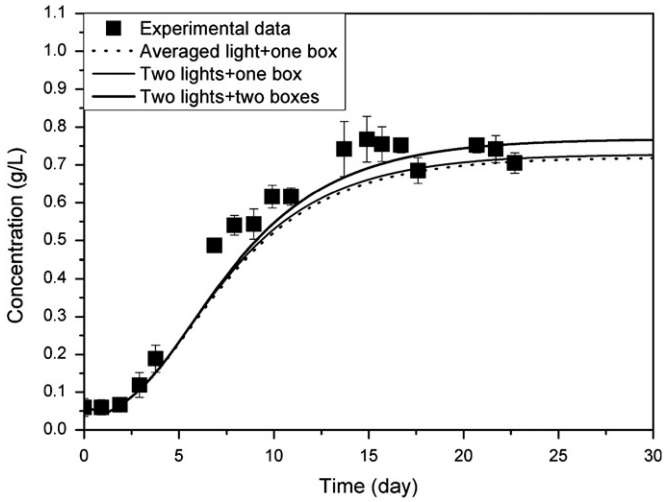

Fig. 9. Comparison of the predicted concentration kinetics with experimental data by Muller-Feuga et al. (2003) for continuous cultures of *P. cruentum*.

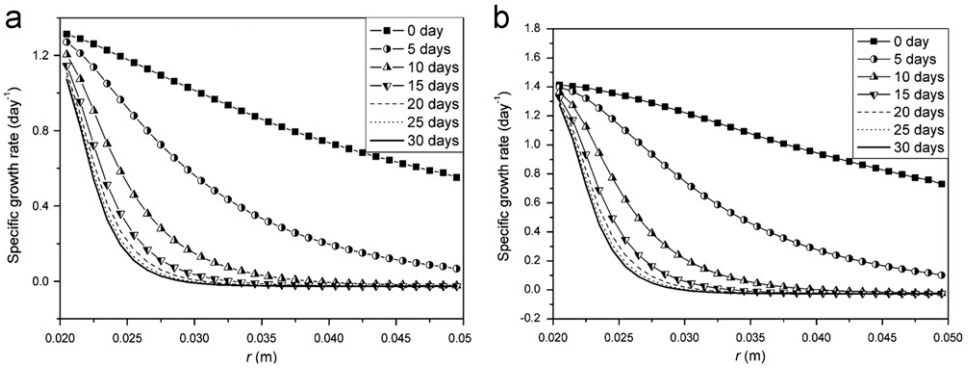

Fig. 8. Dependence of the specific growth rate in the batch culture with the depth in different light sources. (a) Grolux-type tube and (b) Satin-type tube.

kind of PBR and substantial improvements should be made in designing this type of reactors.

The profiles of the specific growth rates in the middle of the reactor with two light sources are displayed in Fig. 12. It is evident that the specific growth rate is dependent on its location in the reactor and the time of cultivation. The phenomenon of severe photolimitation is observed in the reactor only five days after the inoculation. It should be noted that this situation can be alleviated to some extent by either enhancing the light intensity of the sources until the PFD reaches the saturation, increasing the source at the outer region, or using some photoguide devices to maintain as much uniformity in the temporal and spatial distributions as possible. Moreover, it can also be relieved by using a different PBR with low optical thickness after five days of the inoculation. Because the light intensity in this reactor in the whole process is bigger than the value of the compensation light intensity, the outer wall of this PBR even in the last days of cultivation is still in the illumination zone and there is no dark zone in the entire reactor according to the definition utilized in this work, which results in positive specific growth rates in the whole process of cultivation. Under this situation, the correction of L/D ratio does not work and it is equivalent to the case without any correction at all.

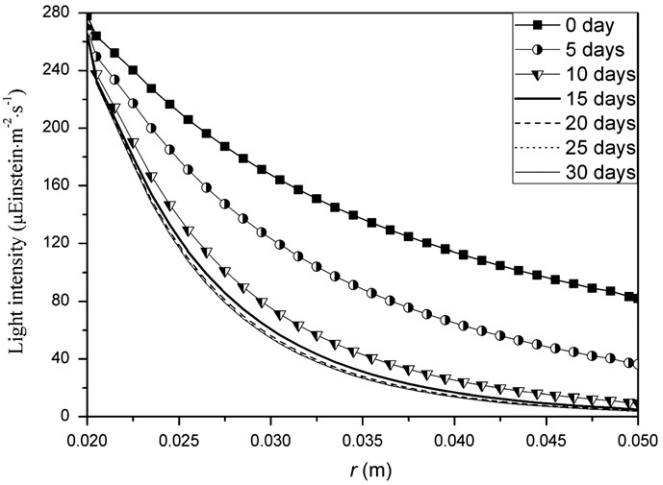

Fig. 10. Evolution of the light intensity with Grolux lamp in *P. cruentum* culture.

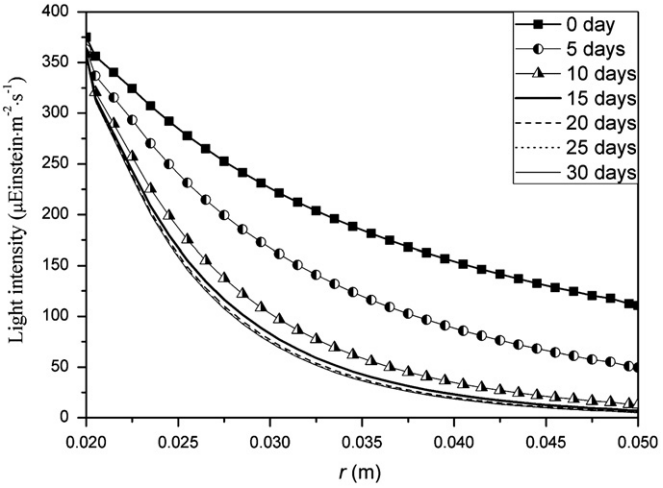

Fig. 11. Evolution of the light intensity with fluorescent lamp in *P. cruentum* culture.

5. Conclusions

An integrated model with radiation transport, photosynthetic growth related to the local instantaneous photosynthetically active irradiance and flow for the cultivation of *P. cruentum* with polychromatic light has been established to predict the microorganism concentration in an annular PBR. The radiation in the reactor is solved by a validated FVM method. A box model, which considers the characteristics of both the spectrum of radiation and extinction coefficients of microalgae, is proposed and adopted to capture the light intensity in the reactor with polychromatic light. Moreover, the effect of the dark zone in the reactor on the light/dark ratio because of the photolimitation is also defined quantitatively to promote the accuracy of the prediction.

The integration of the box model into RTE to predict the polychromatic light transfer is verified first and then two practical applications are illustrated in this work. The predicted concentrations of microalgae are compared to the experimental data for a batch culture and a continuous culture, respectively, with different box models to approximate the two polychromatic lights. The evolution of the light distribution in the PBR is well captured in the present contribution. The results show that a reasonable agreement with the experimental data is observed by using two lights model combined with two boxes and the corrected L/D ratio in both cultures. It can be concluded that these models can be applied to successfully predict the local evolution of light intensity in the PBR. With the help of the developed models, some methods including controlled and optimal light delivery to promote the production of microalgae can be obtained in some elaborate designs. Based on these results, guidelines are also provided to maximize the PBR productivity from the light transport perspective to maintain high light intensity and as much uniformity in its

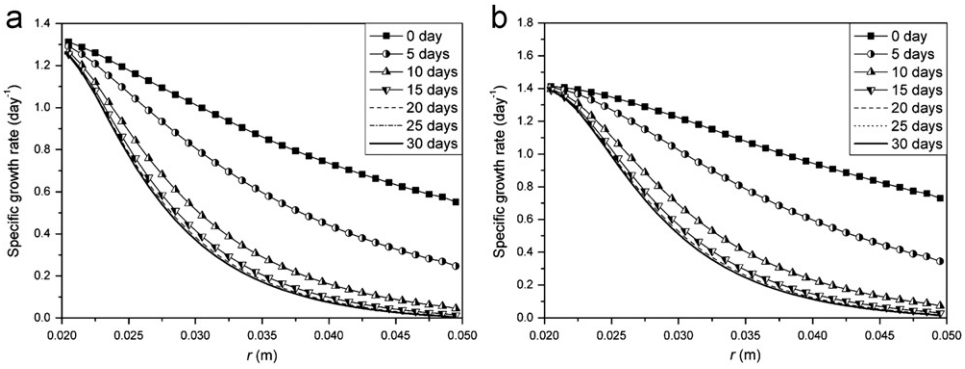

Fig. 12. Dependence of the specific growth rate in the continuous culture with the depth in different light sources. (a) Grolux lamp and (b) fluorescent lamp.

temporal and spatial distribution as possible. The CFD model developed in this work can be used as a tool to model, design and optimize such types of reactors.

Nomenclature

a	absorption coefficient (m^{-1})
D	dilution rate (d^{-1})
G	the incident intensity ($\mu E\ m^{-2}\ s^{-1}$)
$\vec{r}$	position vector (dimensionless)
$\vec{s}_{/}$	direction vector (dimensionless)
$\vec{s}$	scattering director vector (dimensionless)
I	radiation intensity, which depends on position ($\vec{r}$) and direction ($\vec{s}$) ($\mu E\ m^{-2}\ sr^{-1}\ s^{-1}$)
V	volume (m^3)

Greek letters

β	extinction coefficient (m^{-1})
σ	scattering coefficient (m^{-1})
Φ	phase function (dimensionless)
Ω	solid angle in direction $\vec{s}_{/}$(dimensionless)
Ω'	solid angle in direction $\vec{s}$ (dimensionless)
μ	specific growth rate (d^{-1})

Subscripts

au	auxiliary equipments
c	compensation
d	dark
l	light
max	maximum
min	minimum
r	reactor
s	saturation
t	time
total	all the value included
Δt	time step
λ	wavelength
ω	light/dark ratio

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